Relative Humidity Uncertainty Analysis using Dew/Frost Point Measurements

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1. Introduction

Relative Humidity, at any given temperature, can be determined by the ratio of Saturation Vapor Pressure at the dew/frost point temperature to the Saturation Vapor Pressure at the air temperature. In accordance with NIST Guideline 1297[1], uncertainty is determined by analyzing the uncertainty of the individual components, then combining those uncertainties to obtain the total expanded uncertainty. As it applies here, we'll analyze the uncertainties associated with the dew/frost point measurement and the air temperature measurement, then combine them to obtain the total expanded uncertainty in RH. In this analysis, we assume manufacturers stated specifications of ±0.1°C dew/frost point accuracy, and ±0.1°C air temperature measurement accuracy. If these values differ from those given for a specific instrument, then this document can be used as a guide in recomputing the uncertainty due to the stated specifications.

2. Defining Equation

Relative Humidity is determined from dew/frost point measurements by the following equation

$$\%RH = e(t_d) / e(t) * 100 \tag{1}$$

where $e(t_d)$ is the Saturation Vapor Pressure computed at the dew/frost point temperature, t_d,
$e(t)$ is the Saturation Vapor Pressure computed at the air temperature, t,
t_d is the measured dew/frost point temperature, and
t is the measured air temperature.

3. Uncertainty Components

In the mathematical analysis of this formula, we'll analyze the uncertainties due to each of the individual components, then combine the uncertainties to obtain the total expanded uncertainty. We are therefore concerned with the following two uncertainty components.

- uncertainty contribution from the dew/frost point measurement, t_d
- uncertainty contribution from the air temperature measurement, t

3.1 Standard Deviation of t_d and t

We assume a manufacturer's stated accuracy specification of ±0.1°C dew/frost point. In this stated accuracy, we also assume that all uncertainties are accounted for, including measurement uncertainty, measurement resolution, hysteresis, resolution, and self-heating. Based on a rectangular probability distribution, we obtain the standard deviation of t_d

$$\sigma_{td} = 0.1 / \sqrt{3}$$
$$= 0.058$$

Likewise, the standard deviation of the air temperature, t, is obtained

$$\sigma_t = 0.1 / \sqrt{3}$$
$$= 0.058$$

3.2 Uncertainty due to Dew/Frost Point Temperature, t_d

The standard deviation of dew/frost point can now be used to determine the resulting uncertainty in Relative Humidity by the equation

$$u(t_d) = \pm\{RH - e(t_d \pm \sigma_{td}) / e(t) * 100\} \quad (2)$$

3.3 Uncertainty due to Air Temperature, t

The standard deviation of air temperature is used to determine the resulting uncertainty in Relative Humidity by the equation

$$u(t_d) = \pm\{RH - e(t_d) / e(t \pm \sigma_t) * 100\} \quad (3)$$

3.4 Tabulating Individual Uncertainties

Equations 2 and 3 above are used at various temperature and Relative Humidity values, resulting in the following table. For the purposes of this document, we assume that all temperatures are below freezing, and that the Relative Humidity calculations are done in *standard* mode rather than *wmo*.

Notice that at higher humidity, where the ratio of vapor pressures approaches 1, the affect of dew/frost point and air temperature uncertainties increases. Also note that at lower temperatures, the affect of these temperature uncertainties also increases.

Table 1 - Individual Uncertainty Components, $u(t_d)$ & $u(t)$

Temperature	Frost Point	RH	$u(t_d)$	$u(t)$
-15	-18.81	70	0.386	0.374
-15	-17.40	80	0.436	0.428
-15	-16.14	90	0.486	0.481
-15	-15	100	0.535	0.535
-20	-23.66	70	0.401	0.390
-20	-22.30	80	0.453	0.445
-20	-21.09	90	0.505	0.501
-20	-20	100	0.557	0.557
-30	-33.38	70	0.434	0.422
-30	-32.13	80	0.491	0.482
-30	-31.01	90	0.547	0.543
-30	-30	100	0.604	0.604
-40	-43.11	70	0.472	0.459
-40	-41.96	80	0.533	0.525
-40	-40.93	90	0.595	0.590
-40	-40	100	0.656	0.656

4. Combined Standard Uncertainty

The combined standard uncertainty, $u_c(RH)$, is obtained by statistical combination of the standard uncertainty components $u(t_d)$ and $u(t)$. The combined uncertainty is the sum of the variances

$$u_c^2(RH) = u^2(t_d) + u^2(t) \qquad (4)$$

The combined standard uncertainty, $u_c(RH)$, is computed and placed in the following table.

Table 2 - Combined Standard Uncertainty, $u_c(RH)$

Temperature	70% RH	80% RH	90% RH	100% RH
-15	0.537	0.611	0.684	0.757
-20	0.559	0.635	0.711	0.788
-30	0.605	0.688	0.771	0.854
-40	0.658	0.748	0.838	0.928

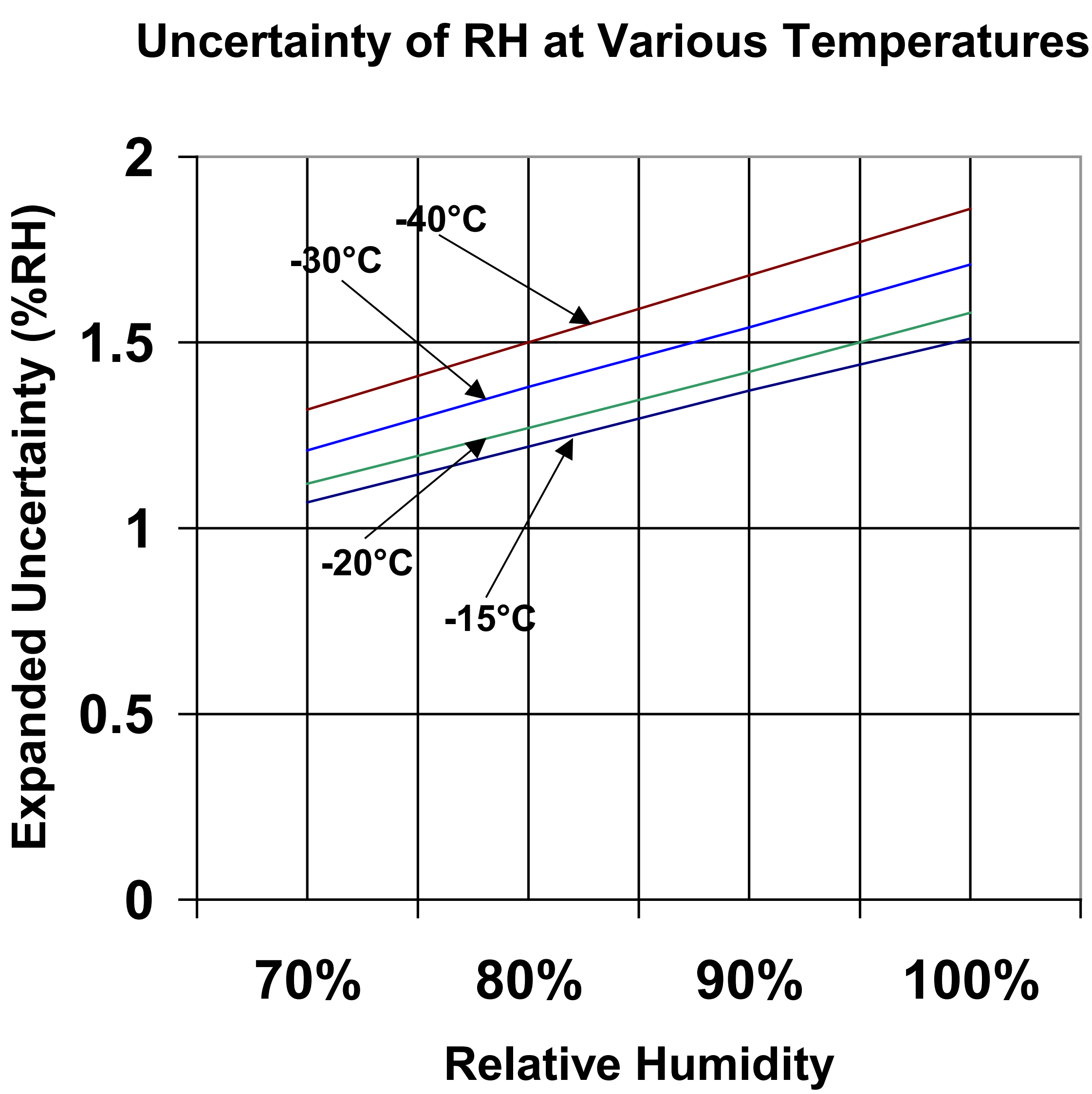
5. Expanded Uncertainty

Utilizing a coverage factor k=2, the expanded uncertainty, U, is expressed in the following table at the various temperatures and humidities, using the formula

$$U = k * u_c(RH) \qquad (5)$$

Expanded Uncertainty, $u_c(RH)$, with coverage factor k=2

Temperature	70% RH	80% RH	90% RH	100% RH
-15°C	±1.07	±1.22	±1.37	±1.51
-20°C	±1.12	±1.27	±1.42	±1.58
-30°C	±1.21	±1.38	±1.54	±1.71
-40°C	±1.32	±1.50	±1.68	±1.86



Reference:

1. Taylor, Barry N. and Kuyatt, Chris, E., *Guidelines for the Evaluation and Expressing theUncertainty of NIST Measurement Results*, NIST Technical Note 1297, 1994 Edition

